To simplify the notations, we shall work in the (sufficiently typical) case $n = 3$. Suppose $\varepsilon : k \to k$ is an anti-automorphism (resp. involution). Composing the transpose map with $\varepsilon$ on matrix entries, we can define $\delta_0 : A \to A$ with

$$\delta_0 \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} \varepsilon(a) & \varepsilon(d) & \varepsilon(g) \\ \varepsilon(b) & \varepsilon(e) & \varepsilon(h) \\ \varepsilon(c) & \varepsilon(f) & \varepsilon(i) \end{pmatrix}$$

It is easy to check that this $\delta_0$ is an anti-automorphism (resp. involution) of $A$, and therefore so is $\delta := \alpha \circ \delta_0$ given by

$$\delta \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} \varepsilon(i) & \varepsilon(f) & \varepsilon(c) \\ \varepsilon(h) & \varepsilon(e) & \varepsilon(b) \\ \varepsilon(g) & \varepsilon(d) & \varepsilon(a) \end{pmatrix}$$

By inspection, we see that this $\delta$ restricts to anti-automorphisms (resp. involutions) on the subrings $R$ and $S$ of $A$.