Label the elements of R as $x_1, x_2, \ldots, x_k$. Since there are at most k distinct elements in the set $\{x_1, x_1^2, \ldots\}$, there must exist
$r_1 < r_2 < \cdots$ such that
$x_1^{r1} = x_1^{r2} = \cdots$.
By considering $\{x_2^{r1}, x_2^{r2}, \ldots\}$, we see similarly that there exist a subsequence $s_1 < s_2 < \cdots$ of $\{r_i\}$ such that
$x_2^{s1} = x_2^{s2} = \cdots$.
Repeating this construction a finite number of times, we arrive at a sequence
$n_1 < n_2 < \ldots$ such that
$x_i^{n1} = x_i^{n2} = \cdots$ for $1 \leq i \leq k$.