

For any nonzero  $a$  we consider descending chain  $aR \supset a^2R \supset \dots$ . It must stop since  $R$  is artinian. So  $a^nR = a^{n+1}R$  for some  $n$ . Then  $a^n = a^{n+1}r$  for some  $r$  in  $R$ .  $a^n(1 - ar) = 0$ . As  $a$  is nonzero, it have to be  $ar = 1$ . Analogously element  $a$  have left inverse. So it is division ring.