Conditions

Use Bisection method to calculate the first root for :
f(d)= 257d^2-640d=0
a=0.0000 b=5.0000
Tolerance=0.0500

Solution

The **bisection method** in mathematics is a root-finding method which repeatedly bisects an interval and then selects a subinterval in which a root must lie for further processing. It is a very simple and robust method, but it is also relatively slow. Because of this, it is often used to obtain a rough approximation to a solution which is then used as a starting point for more rapidly converging methods. The method is also called the **binary search method** or the **dichotomy method**

The method is applicable when we wish to solve the equation $f(x) = 0$ for the real variable x, where f is a continuous function defined on an interval $[a, b]$ and $f(a)$ and $f(b)$ have opposite signs. In this case a and b are said to bracket a root since, by the intermediate value theorem, the f must have at least one root in the interval (a, b).

At each step the method divides the interval in two by computing the midpoint $c = (a+b) / 2$ of the interval and the value of the function $f(c)$ at that point. Unless c is itself a root (which is very unlikely, but possible) there are now two possibilities: either $f(a)$ and $f(c)$ have opposite signs and bracket a root, or $f(c)$ and $f(b)$ have opposite signs and bracket a root. The method selects the subinterval that is a bracket as a new interval to be used in the next step. In this way the interval that contains a zero of f is reduced in width by 50% at each step. The process is continued until the interval is sufficiently small.

Explicitly, if $f(a)$ and $f(c)$ are opposite signs, then the method sets c as the new value for b, and if $f(b)$ and $f(c)$ are opposite signs then the method sets c as the new a. (If $f(c)=0$ then c may be taken as the solution and the process stops.) In both cases, the new $f(a)$ and $f(b)$ have opposite signs, so the method is applicable to this smaller interval.

$$f(a) = f(0) = 0$$

$$f(b) = f(5) = 3225$$

Because the function is continuous, there must be a root within the interval. (Actually, it's obvious to notice, that the root is x=640/257≈2,4902723735408560311284046692607)

In the first iteration, the end points of the interval which brackets the root are $a = 0$ and $b = 5$.

So, $c = \frac{a+b}{2} = 2.5$.

$f(c) = 6.25$

As $f(c) > 0$ and $f(c) < f(b)$ so the next b-point will be c.

New $c = \frac{0+2.5}{2} = 1.25$

$f(c) = -398.438$

As $f(c) < 0$ so the new a is 1.25, the new b is 2.5

New $c = \frac{1.25+2.5}{2} = 1.875$

$f(c) = -296.484$

$a = 1.875$

$b = 2.5$

New $c = \frac{1.875+2.5}{2} = 2.1875$

And so on and so for:

Iteration	an	bn	cn	f(cn)
1	0	5	2,5	6,25
2	0	2,5	1,25	-398,4375
3	1,25	2,5	1,875	-296,484375
4	1,875	2,5	2,1875	-170,2148438
5	2,1875	2,5	2,34375	-88,25683594
6	2,34375	2,5	2,421875	-42,57202148
7	2,421875	2,5	2,4609375	-18,55316162
8	2,4609375	2,5	2,48046875	-6,24961853
9	2,48046875	2,5	2,490234375	-0,024318695

As the tolerance was 0.05, we have a root x = **2,490234375**

We can compare this value with a value 2,4902723735408560311284046692607, which was noticed before. As we see, the calculator's value is very close to the value, given by Bisection method.