

$$A = \liminf x_n = \lim_{n \rightarrow \infty} \left(\inf_{m \geq n} x_m \right)$$

$$a_n = \inf_{m \geq n} x_m$$

$$1) \forall m : a_n \leq x_m$$

$$2) \forall e > 0 \exists m : x_m < a_n + e$$

$$\forall e > 0 \exists N \in \mathbb{N} \forall n > N : |a_n - A| < e \Rightarrow -e < a_n - A < e \Rightarrow A - e < a_n$$

$$\forall n : a_n \leq a_{n+1} \leq \dots \leq A$$

$$A = \sup a_n = \sup_n \left(\inf_{m \geq n} x_m \right) = \liminf x_n$$