We may assume $I \neq 0$, and fix a monic polynomial of the least degree in I. By the usual Euclidean algorithm argument, we see that $I = R \cdot h$. For any $a \in K$, we have $ha \in I = R \cdot h$, so $ha = rh$ for some $r \in R$. By comparing the leading terms, we see that $r \in K$, and in fact $r = a$. Thus, $ha = ah$ for any $a \in K$, which means that $h \in k[x]$