Since $x^2 + 1 \in Z(A)$, $A \cdot (x^2 + 1)$ is an ideal, so we can form the quotient ring $A = A/A \cdot (x^2 + 1)$.
Expressing the ring of real quaternions in the form H = C $\oplus$ Cj, we can define $\varphi : A \to$ H by $\varphi(x) = j$, and $\varphi(a) = a$ for all $a \in$ C. Since $ja = \sigma(a)j$ in H for any $a \in$ C, φ gives a ring homomorphism from A to H. This induces a ring homomorphism $\overline{\phi} : \overline{A} \to$ H, since $\varphi(x^2 + 1) = j^2 + 1 = 0$. In view of $\overline{\phi}\left(\overline{a+bx}\right) = a + bj$ (for $a, b \in$ C), $\overline{\phi}$ is clearly an isomorphism.