

Suppose we have already constructed  $n$  distinct right inverses  $b_1, \dots, b_n$  for  $a$ . We shall show that there exist at least  $n + 1$  distinct right inverses for  $a$ . Indeed, consider the elements  $c_i = 1 - b_i a$  ( $1 \leq i \leq n$ ), which have the property that  $a c_i = a(1 - b_i a) = 0$ . If  $c_i = c_j$ , then  $b_i a = b_j a$ , and right multiplication by  $b_1$  shows that  $b_i = b_j$ . This shows that  $c_1, \dots, c_n$  are distinct. Also, each  $c_i \neq 0$ , since  $a$  has no left inverse. Therefore,  $\{b_1, b_1 + c_1, \dots, b_1 + c_n\}$  are  $n + 1$  distinct right inverses for  $a$ .