Consider any nonzero element $b \in A$, and let $a_n b^n + ... + a_m b^m = 0 \left(a_i \in k,\ a_n \neq 0 \neq a_m,\ n \geq m\right)$ be a polynomial of smallest degree satisfied by b. If $m = 0$, then, for $d = a_n b^{n-1} + ... + a_1$ we have $db = bd = -a_0 \in k^*$. In this case, b is a unit in A. If $b \in B$ and b is a unit in A, the above also shows that $b^{-1} = -a_0^{-1} d \in k\left[b\right] \subseteq B$