

Suppose ${}_R R$ is hopfian, and suppose $ab = 1$. Then $x \rightarrow xb$ defines a surjective endomorphism α of ${}_R R$. Therefore, α is an automorphism. Since $\alpha(ba) = bab = b = \alpha(1)$, we must have $ba = 1$, so R is Dedekind-finite. The converse is proved by reversing this argument.