Conditions

sqrt(x^2)=|x| how can i proof it by real analysis

Solution

To prove this we must consider the definitions of square root and modulus.

In mathematics, a square root of a number a is a number y such that $y^2 = a$, or, in other words, a number y whose square (the result of multiplying the number by itself, or y × y) is a.[1] For example, 4 is a square root of 16 because $4^2 = 16$.

In mathematics, the absolute value (or modulus) | a | of a real number *a* is the non-negative value of *a* without regard to its sign. Namely, | a | = a for a positive *a*, | a | = −a for a negative *a*, and | 0 | = 0. For example, the absolute value of 3 is 3, and the absolute value of −3 is also 3. The absolute value of a number may be thought of as its distance from zero.

Let's fix x, $x \in R$

1) If $x = 0$, then by property of a square root, $\sqrt{0} = 0 = |0|$
2) If $x \geq 0$, then by property of a square root, Since the square-root notation without sign represents the **positive** square root, it follows that $\sqrt{x^2} = x = |x|$.
3) If $x \leq 0$, then by property of a square root, Since the square-root notation without sign represents the **positive** square root, it follows that $\sqrt{x^2} = -x = |x|$.

As we see, for all $x \in R$ the domains and ranges of these functions are identical. This means, that the proposition is proved.