

$$x' = -x/2 + 2x(t-1)e^{-5x(t-1)}, t \in [0,1]$$

$$x(t) = 1, t \in [-1,0]$$

$$t \in [0,1] \Rightarrow t-1 \in [-1,0] \Rightarrow x(t-1) = 1, t \in [0,1]$$

$$x(0) = 1$$

$$\begin{cases} x' = -x/2 + 2e^{-5} \\ x(0) = 1 \end{cases}$$

$$1) x' = -x/2$$

$$\frac{dx}{x} = -(1/2)dt \Rightarrow x = Ce^{-t/2}$$

$$2) x = g(x)e^{-t/2} \Rightarrow x' = g'(x)e^{-t/2} - g(x)e^{-t/2}/2$$

$$g'(x)e^{-t/2} - g(x)e^{-t/2}/2 = -g(x)e^{-t/2}/2 + 2e^{-5}$$

$$g'(x) = 2e^{\frac{t}{2}-5} \Rightarrow g(x) = 2e^{-5} \int e^{t/2} dt = 4e^{t/2-5}$$

$$x = g(x)e^{-t/2} = 4e^{-5}$$

$$3) x = Ce^{-t/2} + 4e^{-5}$$

$$x(0) = 1 \Rightarrow 1 = C + 4e^{-5} \Rightarrow C = 1 - 4e^{-5}$$

$$x = (1 - 4e^{-5})e^{-t/2} + 4e^{-5}$$