

Let E_{ij} be the matrix units. If $r \in Z(R)$, then $(r \cdot I_n)(aE_{ij}) = raE_{ij} = (aE_{ij})(rI_n)$, so $r \cdot I_n \in Z(S)$, where $S = M_n(R)$. Conversely, consider $M = \sum r_{ij}E_{ij} \in Z(S)$. From $ME_{kk} = E_{kk}M$, we see easily that M is a diagonal matrix. This and $ME_{kl} = E_{kl}M$ together imply that $r_{kk} = r_{ll}$ for all k, l , so $M = r \cdot I_n$ for some $r \in R$. Since this commutes with all $a \cdot I_n$ ($a \in R$), we must have $r \in Z(R)$.