Let $I_i = I \cap B_i$, and write $B_i = Re_i$. We claim that $\oplus I_i \subseteq I$ is an equality. Indeed, for any $a \in I$, we have $a = 1a = e_1a + \cdots + e_na$ with $e_ia = ae_i \in I \cap B_i = I_i$. Therefore, $I = \oplus I_i$, where I_i is a left ideal of R (and also of B_i). If I is an ideal of R, clearly I_i is an ideal of R (and also of B_i).