

Let  $R$  be the ring of  $2 \times 2$  upper triangular matrices over a nonzero ring  $k$ , and let  $x = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ . A simple calculation shows that  $Rx = \begin{pmatrix} k & 0 \\ 0 & 0 \end{pmatrix}$ , and  $xR = \begin{pmatrix} k & k \\ 0 & 0 \end{pmatrix}$ . Therefore,  $Rx \subsetneq xR$ . Alternatively, we can take  $R$  to be any non-Dedekind – finite ring, say with  $xy = 1 \neq yx$ . Then  $xR$  contains 1 so  $xR = R$ , but  $1 \notin Rx$  implies that  $Rx \subsetneq R = xR$ .