Prove that there exist real numbers which are not algebraic.

A complex number z is said to be algebraic if there are integers $a_0, \dots, a_n$, not all zero, such that

$$a_0 z^n + a_1 z^{n-1} + \dots + a_{n-1^z} + a_n = 0$$

Prove that the set of all algebraic numbers is countable. Hint: For every positive integer N there are only finitely many equations with

$n + |a_0| + |a_1| + \dots + |a_n| = N.$

Proof: For every positive integer N there are only finitely many equations with

$n + |a_0| + |a_1| + \dots + |a_n| = N.$

(since 1 ≤ n ≤ N and 0 ≤ $|a_0|$ ≤ N). We collect those equations as C_N . Hence ∪ C_N is countable. For each algebraic number, we can form an equation and this equation lies in C_M for some M and thus the set of all algebraic numbers is countable.

Proof: *If not,* R^1 *= { all algebraic numbers } is countable, a contradiction.*