A= 2X+Y+3Z
B= 3X-2Z
Find a vector C whose magnitude is 6 and whose direction is perpendicular to A and B

Solution

Let c = (a,b,c)

$$6 = (a^2 + b^2 + c^2)^{\frac{1}{2}}$$

since direction(d) of c is perpendicular to both A and B
then
the A . d=1--> $2a + b + 3c = 1$
B . d=1--> $3a - 2c = 1 \gg a = \frac{1+2c}{3} \gg 2\frac{1+2c}{3} + b + 3c = 1 \gg 13c + 3b = 1$

$$\gg b = \frac{1 - 13c}{3}$$

then
$36 = \left((\frac{1+2c}{3})^2 + (\frac{1-13c}{3})^2 + c^2\right) \gg 182c^2 - 22c - 322 = 0 \gg c_1 = 1.4, c_2 = -1.3$

$$a_1 = \frac{1 + 2 * 1.4}{3} = 1.3, a_2 = \frac{1 + 2 * (-1.3)}{3} = -0{,}5$$

$$b_1 = \frac{1 - 13 * 1.4}{3} = -5{,}7, b_2 = \frac{1 + 13 * 1.3}{3} = 6$$

Answer: $(1.3, -5.7, 1.4), (-0.5, 6, -1.3)$.