Question #16379

Let us have a function $f(y(x), x)=0$. It implicitly gives $y=y(x)$. Taking the derivative and using the chain rule, obtain: $\frac{\partial f}{\partial y}\frac{dy}{dx}+\frac{\partial f}{\partial x}=0 \Rightarrow \frac{dy}{dx}=-\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$.

For our case, lets rewrite given equality as $f(x, y)=x^y \cdot y^x - m=0$. Then,

$$\frac{\partial f}{\partial x}=y^{x+1}x^{y-1}+x^y y^x \ln y, \frac{\partial f}{\partial y}=x^{y+1}y^{x-1}+y^x x^y \ln x$$, so $$\frac{dy}{dx}=-\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}=\frac{-(x^y \ln y+y x^{y-1})}{(x^{y+1}y^{-1}+x^y \ln x)}$$.