

Say $P \oplus Q = F$, where F is free with basis $\{e_i \mid i \in I\}$, and write $p = e_1 a_1 + \dots + e_n a_n$ ($a_i \in R$). (For convenience, we assume $\{1, 2, \dots, n\} \subset I$). Decompose e_i into $p_i + q_i$, where $p_i \in P, q_i \in Q$. Then $p = p_1 a_1 + \dots + p_n a_n$ implies that $\sum_{i=1}^r R a_i$ cannot be contained in a maximal left ideal m (for otherwise $p \in P m$). Thus, $\sum_{i=1}^r R a_i = R$, so by the Unimodular Column Lemma, pR is free on p , and $pR \oplus X = F$, for a suitable submodule $X \subset F$. But then $pR \oplus (X \cap P) = P$, as desired.