Fix an integrality equation $x^{n+1}+a_n x^n+...+a_0=0$, where $a_i \in R$, and pick a non 0-divisor $r \in R$ such that $rx_i \in R, 1 \le i \le n$. Consider the f.g. ideal $I=\sum_{i=1}^{n} Rrx^i$ in R. By hypothesis, ${}_R I$ is projective, so by the Dual Basis Lemma, there exist linear functionals $f_i : I \to R$ such that $b=\sum_{i=1}^{n} f_i(b) rx^i$ for all $b \in I$. Now from the integrality equation, we have $rx^{i+1} \in I$, so we can write

$$rx=\sum_{i=1}^{n} f_i(rx) rx^i=\sum_{i=1}^{n} f_i\left(r^2 x^{i+1}\right)=r\sum_{i=1}^{n} f_i\left(rx^{i+1}\right)$$

Cancelling r, we conclude that $x=\sum_{i=1}^{n} f_i\left(rx^{i+1}\right) \in R$.