

Say $P = P = Rx_1 \oplus \dots \oplus Rx_n$, where $x_i \in P$. Let $x = x_1 + \dots + x_n$, and $Q = Rx \subseteq P$. Consider any prime ideal $p \subset P$. Since $P_p \cong R_p$ and R_p is indecomposable, we must have $(Rx_i)_p = 0$ for all but one i , and hence $P_p = Q_p$. Since this holds for all primes p , we have $P = Q$. Therefore, the map $R \rightarrow P$ sending 1 to x is a split surjection, which must then be an isomorphism since $\text{rk } P = 1$.