Since P can be embedded in a free module, we may as well assume that P is free. Since M is f.g., we may also assume that $P \cong R^n$ for some finite n. Suppose M can be generated by m elements. By adding copies of R to P, we may assume that n $\geq$ m. Then there exists $f \in End_R(P)$ with f(P) = M. Now

$$End_R(P) \cong End_R(R^n) \cong M_n(R)$$

is a von Neumann regular ring. Therefore, there exists $g \in End_R(P)$ such that f = fgf. Then e = fg is an idempotent endomorphism of P. We have $f(P) = fgf(P) \subseteq e(P) \subseteq f(P)$, so M = f(P) = e(P) is a direct summand of P, with direct complement $(1-e)(P)$