

Let  $m_1, \dots, m_n$  be all maximal ideals, that contain  $I$ .

As in the solution to Ex. 2.11B, there exists an element  $b \in I \setminus \bigcup_{i=1}^n m_i I$ . The ideal  $I^{-1}b$  cannot be contained in any  $m_i$  (since  $I^{-1}b \subseteq m_i$  would imply that  $b \in m_i I$ , by the invertibility of  $I$ ). Therefore,  $I + I^{-1}b$  is contained in no maximal ideals of  $R$ , and so  $I + I^{-1}b = R$ . Multiplying this equation by  $I$ , we get  $I^2 + Rb = I$ . Now the ideal  $I/Rb$  in the ring  $R/Rb$  is f.g. and idempotent. This implies that  $I/Rb$  is generated by a suitable (idempotent) element  $a + Rb$  in  $R/Rb$ . From this, we have clearly  $I = Ra + Rb$ .