

Consider the exact sequence of right R -modules $0 \rightarrow \text{ann}_r(a) \rightarrow R \xrightarrow{f} aR \rightarrow 0$ where f is defined by left multiplication by a . If aR is projective, this sequence splits. Then $\text{ann}_r(a)$ is a direct summand of R_R , so $\text{ann}_r(a) = eR$ for some $e^2 = e \in R$. Conversely, if $\text{ann}_r(a) = eR$ where $e^2 = e \in R$, then the direct sum decomposition $R = eR \oplus (1-e)R$ implies that $aR \cong R/eR \cong (1-e)R$, which is a projective right R -module.