R is not Dedekind-finite iff there exists an R-isomorphism $R_R \cong R_R \oplus X$ for some R-module $X \neq 0$. This means that $R = eR \oplus (1-e)R$ for some idempotent $e \neq 1$ such that $(eR)_R \cong R_R$.