

The answer is "no." Let $V = e_1k \oplus e_2k \oplus \dots$ where k is any division ring. Since V_k is a semisimple module, $R = \text{End}(V_k)$ is a von Neumann regular ring. Let $x, y \in R$ be defined by $y(e_i) = e_{i+1}$, $x(e_i) = e_{i-1}$, where $i \geq 1$ and e_0 is taken to be 0. Then $xy = 1 \in R$, but $yx \neq 1$ (since $yx(e_1) = 0$). Therefore, R is not von Neumann finite.