

The answer is "no." Let  $V = e_1k \oplus e_2k \oplus \dots$  where  $k$  is any division ring. Since  $V_k$  is a semisimple module,  $R = \text{End}(V_k)$  is a von Neumann regular ring. Let  $x, y \in R$  be defined by  $y(e_i) = e_{i+1}$ ,  $x(e_i) = e_{i-1}$ , where  $i \geq 1$  and  $e_0$  is taken to be 0. Then  $xy = 1 \in R$ , but  $yx \neq 1$  (since  $yx(e_1) = 0$ ). Therefore,  $R$  is not von Neumann finite.