

Assume, for the moment, that $g(M) \subset M$, $g(M) \neq M$. Since g is an injection,

$g^2(M) \subset g(M)$, $g^2(M) \neq g(M)$. Repeating this argument, we see that $g^{n+1}(M) \subset g^n(M)$, $g^{n+1}(M) \neq g^n(M)$ for all n . Therefore, we have a strictly descending chain

$$M \supset g(M) \supset g^2(M) \supset \dots$$

contradicting the fact that M is an artinian module.