

Assume, for the moment, that  $g(M) \subset M, g(M) \neq M$ . Since  $g$  is an injection,  $g^2(M) \subset g(M), g^2(M) \neq g(M)$ . Repeating this argument, we see that  $g^{n+1}(M) \subset g^n(M), g^{n+1}(M) \neq g^n(M)$  for all  $n$ . Therefore, we have a strictly descending chain

$$M \supset g(M) \supset g^2(M) \supset \dots$$

contradicting the fact that  $M$  is an artinian module.