Question #16146 Determine whether the given differential equation is exact. If it is exact, solve it. $(5x + 4y)dx + (4x - 8y^3)dy = 0$. .

Solution. We are to verify $\dfrac{\partial(5x + 4y)}{\partial y} = \dfrac{\partial(4x - 8y^3)}{\partial x}$. Hence the differential equation is exact.

To solve it, write $\dfrac{\partial U(x,y)}{\partial x} = 5x + 4y$, thus $U(x,y) = 5/2x^2 + 4yx + \varphi(y)$.

Next, $\dfrac{\partial U(x,y)}{\partial y} = 4x - 8y^3$ and $4x + \varphi'(y) = 4x - 8y^3$, so $\varphi'(y) = -8y^3$ or $\varphi(y) = -2y^4 + C$. To conclude the general solution $U(x,y) = C$ or equivalently $5/2x^2 + 4yx - 2y^4 = C$.

Solution. $5/2x^2 + 4yx - 2y^4 = C$.