Since $\mathbb{R}$ is Banach space so is $B(X)$ - space of all bounded linear functionals defined on X .
Then:

$$\left\| f_n - f_{n+p} \right\| = \left\| f_n - f_{n+1} + f_{n+1} - f_{n+2} + f_{n+2} - ... + f_{n+p-1} - f_{n+p} \right\| \le c_n + c_{n+1} + ... + c_{n+p-1} \le$$

$$\le \sum_{k=n}^{\infty} c_k \xrightarrow[n\to\infty]{} 0$$

So sequence $(f_n)_{n\in\mathbb{N}}$ is Cauchy sequence, and as $B(X)$ is complete then in converges to some $f \in B(X)$, and so it is uniformly convergent by definition.