

Question #16055

Given, $y(x) = C_1 e^x + e^{C_1} e^{2x} + \ln C_1 e^{3x}$, let's make substitution $e^x = t$. So,

$y(t) = C_1 t + e^{C_1} t^2 + \ln C_1 t^3$. Then, find the derivatives:

$y'(t) = 3t^2 \ln C_1 + 2e^{C_1} t + C_1$; $y''(t) = 6t \ln C_1 + 2e^{C_1}$; $y'''(t) = 6 \ln C_1 \Rightarrow C_1 = e^{\frac{y'''}{6}}$. So, the differential equation is $y(t) = \frac{y'''}{6} t^3 + \exp(\exp(\frac{y'''}{6})) t^2 + \exp(\frac{y'''}{6}) t$.