

For matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

given condition are

$$ad - bc > 0, \quad a + d < 0$$

If we try to find eigenvalues

$$\begin{pmatrix} a - \lambda & b \\ c & d - \lambda \end{pmatrix}$$

$$(a - \lambda)(b - \lambda) - bc = 0$$

$$\lambda^2 - B\lambda + C = 0, \quad B = a + d, C = ad - bc$$

$$\lambda = \frac{B \pm \sqrt{B^2 - 4C}}{2}$$

we will see that both of them are negative.