

$$\begin{aligned}\int_{1/a}^a \frac{1}{x} \sin(x-1/x) dx &= - \int_a^{1/a} \frac{1}{x} \sin(x-1/x) dx = |y = 1/x| = - \int_a^{1/a} y \sin(1/y-y) d\left(\frac{1}{y}\right) = \\ &= \int_{1/a}^a y \sin(y - 1/y) \frac{1}{y^2} dy = - \int_{1/a}^a \frac{1}{y} \sin(y - 1/y) dy = 0\end{aligned}$$