$$\int_{1/a}^{a} \frac{1}{x}\sin(x-1/x)dx = -\int_{a}^{1/a} \frac{1}{x}\sin(x-1/x)dx = |y = 1/x| = -\int_{a}^{1/a} y\sin(1/y-y)d(\frac{1}{y}) =$$

$$= \int_{1/a}^{a} y\sin(y-1/y)\frac{1}{y^2}dy = -\int_{1/a}^{a} \frac{1}{y}\sin(y-1/y)dy = 0$$