Find the length of the line segment described by the function $y = \frac{1}{x}$ between $x = 1$ and $x = 4$. Round your answer to two decimal places.

a. 3.55
b. 3.35
c. 3.15
d. 2.95

Solution: $l = \int_a^b \sqrt{1 + (y')^2}\, dx$

$$l = \int_1^4 \sqrt{1 + (x^{-2})^2}\, dx = \int_1^4 \sqrt{\frac{x^4 + 1}{x^4}}\, dx = \begin{bmatrix} x^2 = sht \\ x = \sqrt{sht} \\ dx = \dfrac{cht\, dt}{2\sqrt{sht}} \end{bmatrix}$$

$$= \int_{x=1}^{x=4} \sqrt{\frac{sh^2t + 1}{sh^2t}} * \frac{cht\, dt}{2\sqrt{sht}} = \int_{x=1}^{x=4} \sqrt{\frac{ch^2t}{sh^2t}} * \frac{cht\, dt}{2\sqrt{sht}}$$

$$= \int_{x=1}^{x=4} \frac{cht}{sht} * cht\, d\left(\sqrt{sht}\right) = \int_{x=1}^{x=4} \frac{1 + sh^2t}{sht}\, d\left(\sqrt{sht}\right) = \left[z = \sqrt{sht}\right]$$

$$= \int_{x=1}^{x=4} \frac{1 + z^4}{z^2}\, dz = \left(-\frac{1}{3z^3} + \frac{z^3}{3}\right)\Big|_{x=1}^{x=4}$$

$$= \left(-\frac{1}{3\left(\sqrt{sht}\right)^3} + \frac{\left(\sqrt{sht}\right)^3}{3}\right)\Big|_{x=1}^{x=4} = \left(-\frac{1}{3x^3} + \frac{x^3}{3}\right)\Big|_{x=1}^{x=4} = 3.15$$

Answer: c. 3.15