Question 1. *Let G be a group. Let H be a finite subset of G and let H be closed with respect to multiplication. Prove that H is a subgroup of G.*

Solution. It is obviously enough to prove that the identity $e \in H$ and that for any $h \in H$ the inverse $h^{-1} \in H$.

Consider an arbitrary $h \in H$. Since H is closed under multiplication, we conclude that all the powers $h, h^2, h^3, \ldots$ belong to H. But H is finite, so there is $n \in \mathbb{N} \cup 0$ such that $h^n = e$. Thus, $e \in H$. If $h \neq e$, then $n > 0$, so $h^{n-1} \in H$. Note that $h \cdot h^{n-1} = h^n = e$, therefore, $h^{-1} = h^{n-1} \in H$. $\square$