$$\lim_{n\to\infty} n\left[\sqrt{n^2+2}-n\right] = \lim_{n\to\infty} \frac{n\left(\sqrt{n^2+2}-n\right)\left(\sqrt{n^2+2}+n\right)}{\sqrt{n^2+1}+n} = \lim_{n\to\infty} \frac{n\left(n^2+2-n^2\right)}{\sqrt{n^2+2}+n} =$$

$$= \lim_{n\to\infty} \frac{2n}{\sqrt{n^2+2}+n} = \lim_{n\to\infty} \frac{2}{\sqrt{1/n^2+2}+1} = \frac{2}{\sqrt{2}+1}$$