

$$\begin{aligned}\lim_{n \rightarrow \infty} n \left[\sqrt{n^2 + 2} - n \right] &= \lim_{n \rightarrow \infty} \frac{n \left(\sqrt{n^2 + 2} - n \right) \left(\sqrt{n^2 + 2} + n \right)}{\sqrt{n^2 + 2} + n} = \lim_{n \rightarrow \infty} \frac{n \left(n^2 + 2 - n^2 \right)}{\sqrt{n^2 + 2} + n} = \\ &= \lim_{n \rightarrow \infty} \frac{2n}{\sqrt{n^2 + 2} + n} = \lim_{n \rightarrow \infty} \frac{2}{\sqrt{1/n^2 + 2} + 1} = \frac{2}{\sqrt{2} + 1}\end{aligned}$$