

1) Associativity

$$\begin{pmatrix} a \\ b \end{pmatrix} * \left( \begin{pmatrix} c \\ d \end{pmatrix} * \begin{pmatrix} e \\ f \end{pmatrix} \right) = \begin{pmatrix} a \\ b \end{pmatrix} * \begin{pmatrix} c + de \\ df \end{pmatrix} = \begin{pmatrix} a + bc + bde \\ bdf \end{pmatrix}$$

$$\left( \begin{pmatrix} a \\ b \end{pmatrix} * \begin{pmatrix} c \\ d \end{pmatrix} \right) * \begin{pmatrix} e \\ f \end{pmatrix} = \begin{pmatrix} a + bc \\ bd \end{pmatrix} * \begin{pmatrix} e \\ f \end{pmatrix} = \begin{pmatrix} a + bc + bde \\ bdf \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} * \left( \begin{pmatrix} c \\ d \end{pmatrix} * \begin{pmatrix} e \\ f \end{pmatrix} \right) = \left( \begin{pmatrix} a \\ b \end{pmatrix} * \begin{pmatrix} c \\ d \end{pmatrix} \right) * \begin{pmatrix} e \\ f \end{pmatrix}$$

2) Identity

$$\begin{pmatrix} a \\ b \end{pmatrix} * \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} * \begin{pmatrix} a \\ b \end{pmatrix}$$

3) Inverse:

$$\begin{pmatrix} a \\ b \end{pmatrix} * \begin{pmatrix} -a/b \\ 1/b \end{pmatrix} = \begin{pmatrix} -a/b \\ 1/b \end{pmatrix} * \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

4) Commutativity:

$$\begin{pmatrix} a \\ b \end{pmatrix} * \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a + bc \\ bd \end{pmatrix}$$

$$\begin{pmatrix} c \\ d \end{pmatrix} * \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} c + ad \\ db \end{pmatrix}$$

$$a + bc \neq c + ad$$

So it is non-commutative group.