

## Conditions

Four points are given by their coordinates in a rectangular Cartesian system: A(1,0,2); B(0,1,3); C(1,1,1); and D(1,1,4). Evaluate the area of triangle ABD

## Solution

Let's construct vectors  $\overline{AB}$ ,  $\overline{BD}$ ,  $\overline{AD}$ :

$$\overline{AB} = (0 - 1; 1 - 0; 3 - 2) = (-1, 1, 1)$$

$$\overline{BD} = (1 - 0; 1 - 1; 4 - 3) = (1, 1, 1)$$

$$\overline{AD} = (1 - 1; 1 - 0; 4 - 2) = (0, 1, 2)$$

Now let's find vectors length:

$$|\overline{AB}| = \sqrt{(-1)^2 + 1^2 + 1^2} = \sqrt{3}$$

$$|\overline{BD}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$|\overline{AD}| = \sqrt{0^2 + 1^2 + 2^2} = \sqrt{5}$$

The area is:

$$S = \sqrt{p(p - |\overline{AB}|)(p - |\overline{BD}|)(p - |\overline{AD}|)}$$

$$\text{where } p = \frac{(|\overline{AB}| + |\overline{BD}| + |\overline{AD}|)}{2} = \sqrt{3} + \frac{\sqrt{5}}{2}$$

$$\begin{aligned} S &= \sqrt{(\sqrt{3} + \frac{\sqrt{5}}{2})(\frac{\sqrt{5}}{2})(\frac{\sqrt{5}}{2})(\sqrt{3} + \frac{\sqrt{5}}{2} - \frac{\sqrt{5}}{2})} = \sqrt{(\sqrt{3} + \frac{\sqrt{5}}{2})(\frac{\sqrt{5}}{2})(\frac{\sqrt{5}}{2})(\sqrt{3} - \frac{\sqrt{5}}{2})} \\ &= \sqrt{\left(3 - \frac{5}{4}\right)\left(\frac{5}{4}\right)} = \sqrt{\frac{7 \cdot 5}{4 \cdot 4}} = \frac{\sqrt{35}}{4} \end{aligned}$$

## Answer

$$\frac{\sqrt{35}}{4}$$