

$$f : X \rightarrow Y$$

$$(\forall \varepsilon > 0)(\exists \delta > 0)(\forall x_1, x_2)[d_X(x_1, x_2) < \delta \Rightarrow d_Y(f(x_1), f(x_2)) < \varepsilon]$$

$$\{x_n\}_{n=1}^{\infty} : (\forall \varepsilon > 0)(\exists N > 0)(\forall n, m \geq N)\{d_X(x_n, x_m) < \varepsilon\}$$

$$\delta := \varepsilon \Rightarrow d_X(x_m, x_n) < \varepsilon \Rightarrow d_Y(f(x_m), f(x_n)) < \varepsilon$$

$$\{f(x_n)\}_{n=1}^{\infty} : (\forall \varepsilon > 0)(\exists N > 0)(\forall n, m \geq N)\{d_X(f(x_n), f(x_m)) < \varepsilon\}$$

So image of Cauchy sequence is again Cauchy sequence, if f is uniformly continuous.

Since every continuous function defined on compact set is uniformly continuous, and complete space is compact, then image of Cauchy sequence is again Cauchy.

But completeness of Y cannot guarantee the same statement as above.

So, a and b are true, and c is false.