Conditions

describe the column space (range) and the nullspace (kernel) of the matrices

A= 1 -1 B= 0 0 0

0 0 0 0 0

Solution

In linear algebra, the column space, C(A) of a matrix (sometimes called the range of a matrix) is the set of all possible linear combinations of its column vectors. The column space of an m × n matrix is a subspace of m-dimensional Euclidean space. The dimension of the column space is called the rank of the matrix. The column space of a matrix is the image or range of the corresponding matrix transformation.

$$A = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$$

Let's consider a linear combination for A:

$$c_1 v_1 + c_2 v_2$$

where c_1, c_2 are scalars. The set of all possible linear combinations of v_1, v_2 is called the column space of A:

$$c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} c_1 \\ 0 \end{pmatrix} + \begin{pmatrix} -c_2 \\ 0 \end{pmatrix} = \begin{pmatrix} c_1 - c_2 \\ 0 \end{pmatrix}$$

In this case, the column space is precisely the set of vectors $(x, 0) \in R^2$ for all $x \in R$

B:

$$B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

It's a zero matrix.

Let's consider a linear combination for B:

$$c_1 \begin{pmatrix} 0 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 0 \end{pmatrix} + c_3 \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

In this case, the column space is null vector $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

In linear algebra, the kernel or null space (also nullspace) of a matrix A is the set of all vectors x for which Ax = 0. The kernel of a matrix with n columns is a linear subspace of n-dimensional Euclidean space. The dimension of the null space of A is called the nullity of A.

A:

$$Ax = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - x_2 \\ 0 \end{pmatrix} = 0$$

$x_1 - x_2 = 0$

$x_1 = x_2$

The kernel of matrix A are all vectors $(x_1, x_2) \in R^2$, where $x_1 = x_2$, $x_2 \in R$

B:

$$Bx = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

This means that the kernel of matrix B are all vectors $(x_1, x_2) \in R^2$, or R^2 itself.

Answer

C(A): $(x, 0) \in R^2$ for all $x \in R$

C(B) is null vector $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Ker(A): $(x_1, x_2) \in R^2$, where $x_1 = x_2$, $x_2 \in R$

Ker(B): R^2