

Question #15429 Calculate the mean, variance, skewness and kurtosis for the probability density function with an 'unit' distribution $[0, 1]$.

Solution. Mean of a $\xi \sim \text{Unif}[0, 1]$ $m = E\xi = \int_0^1 x dx = 1/2$, second moment is $E\xi^2 = \int_0^1 x^2 dx = 1/3$, thus the variance is $\sigma^2 = 1/3 - 1/4 = 1/12$. Next, calculate the third moment $E\xi^3 = \int_0^1 x^3 dx = 1/4$, thus the skewness is $\gamma_1 = \frac{1/4 - 3/2 \cdot 1/12 - 1/8}{(1/12)^{3/2}} = 0$. To calculate kurtosis, first calculate the central fourth moment, clearly it is equals $\int_0^1 (x - 1/2)^4 dx = 2 \int_0^{1/2} x^4 dx = 1/(5 \cdot 2^4)$, next $\gamma_2 = \frac{E\xi^4}{\sigma^4} - 3 = \frac{1/2^4}{5/12^2} - 3 = 144/(5 \cdot 16) - 3 = -6/5 = -1.2$