

Question #15423 Show that the function: $f(x) = [x - 2][3 + \sin(1/(x - 2))]/[1 + x^2]$, $x \neq 2$ and $f(2) = 0$. Show that f is continuous at $x = 2$.

Solution. We are to show that $\lim_{x \rightarrow 2} f(x) = f(2) = 0$. First note that $\lim_{x \rightarrow 2} (x - 2) \sin(1/(x - 2)) = 0$, due to \sin is bounded, $1 + x^2 \rightarrow 5$, $x \rightarrow 2$ and $3(x - 2) \rightarrow 0$, $x \rightarrow 2$ and the result follows from the arithmetic rules for limits.