Question #15407 Carol lives in the east end of Montreal. To get to school for her morning classes Carol has the option of taking the bus, going underground with the metro, or driving her car downtown to get to School. Carol prefers the metro, especially in the winter, and she will opt for it 50% of the time. The bus is perhaps a more convenient option since it stops near her house, so she will choose that option 40% of the time. The car is the most expensive option, but it is the most reliable, as demonstrated by the fact that she is only late for class 5% of the time when she drives. The bus is the least reliable of the options as it gets Carol to class on time 70% of the time, whereas this value increases by 10% with the metro.
a) What is the probability that Carol is late to class on a given morning?
b) What is the probability that Carol took the metro and is on time for class?
c) What is the probability that Carol did not drive and is on time for class? .
Solution. Denote by H_1, H_2, H_3 Carol decision to take bus, netro and car respectively. Then $P(H_1) = 0.4, P(H_2) = 0.5, P(H_3) = 0.1$, also denote by A the event that Carol will be late, so the condition entails $P(A \mid H_1) = 0.3, P(A \mid H_2) = 0.2, P(A \mid H_3) = 0.05$.
a) Due to the formula of total probability $P(A) = P(A \mid H_1)P(H_1) + P(A \mid H_2)P(H_2) + P(A \mid H_3)P(H_3) = 0.12 + 0.1 + 0.005 = 0.2205$.
b) $P(\overline{A} \cap H_2) = P(\overline{A} \mid H_2)P(H_2) = 0.5(1 - 0.2) = 0.4$.
c) $P(\overline{A} \cap \overline{H_3}) = 1 - P(A) - P(H_3) + P(A \cap H_3) = 1 - 0.2205 - 0.1 + 0.05 \cdot 0.1 = 0.68$.