a)

$\forall s(x) \in S : s(x) = 2a(x) + xb(x)$

$s(0) = 2a(0) + 0 = 2a(0) \in 2\mathbb{Z} \Rightarrow s(x) \in \{ f(x) | f(0) \in 2\mathbb{Z} \}$

$\forall s(x) \in \{ f(x) | f(0) \in 2\mathbb{Z} \} \Rightarrow s(x) = 2a_0 + a_1 x + a_2 x^2 + ... + a_n x^n \in S$

$S = \{ f(x) | f(0) \in 2\mathbb{Z} \}$

b)

S can be expressed in form $S = 2\mathbb{Z}[x] + x\mathbb{Z}[x]$, so it is finitely generated ideal (by definition) in domain $\mathbb{Z}[x]$.

c)

If $S = d(x)\mathbb{Z}[x]$ then $d(x)$ have to divide both 2 and x, but only constants are common divisors.

So $d(x) = const$, and only possible constants are 1 and -1. So there is no d(x) with this property.

Z[x] is not PID.