Question #15348 The mean and standard deviation for the quality grade point averages of a random sample of 28 college seniors are calculated to be 2.6 and 0.3 respectively. Find the 95% confidence interval for the mean of the entire senior class. How large a sample is required if we want to be 95% confident that our estimate of μ is not off by more than 0.05?

Solution. Let μ be the mean of the entire senior class. Given: $n = 28$, $s^2 = 0.3$, $\overline{x} = 2.6$, $(1 - \alpha) = 0.95$.

(a) A 95% confidence interval estimate for the is

$$\overline{x} - z_{0.025}s/\sqrt{n} \le \mu \le \overline{x} + z_{0.025}s/\sqrt{n},$$

$$2.6 - 1.96 \cdot 0.3/5.3 \le \mu \le 2.6 + 1.96 \cdot 0.3/5.3$$

or $2.49 \le \mu \le 2.71$.

(b) Let n be the required sample size. To be 95% confident that if off by less than 0.05 would implies $z_{0.025}s/\sqrt{n} < 0.05$, or $n \ge \left[\dfrac{0.3 \cdot 1.96}{0.05} \right]^2 \approx 139$.

Answer. a) $[2.49, 2.71]$, b) 139.