We will prove it by induction:

1. When $n = 0 : \left(2 \cdot 0 + 1\right)^2 = 1 = \frac{\left(0+1\right)\left(2\cdot 0+1\right)\left(2\cdot 0+3\right)}{3} \Rightarrow True.$
2. Let take that if n = k then the given formula is also true. Then we will have:

$$n = k : \sum_{n=0}^{k}\left(2\cdot n+1\right)^2 = \frac{\left(k+1\right)\left(2\cdot k+1\right)\left(2\cdot k+3\right)}{3} \Rightarrow True.$$

3. Let show that if n = k + 1 the formula is true too:

$$n = k+1 : \sum_{n=0}^{k+1}\left(2\cdot n+1\right)^2 = \sum_{n=0}^{k}\left(2\cdot n+1\right)^2 + \left(2\cdot k+3\right)^2 =$$

$$= \frac{\left(k+1\right)\left(2\cdot k+1\right)\left(2\cdot k+3\right)}{3} + \left(2\cdot k+3\right)^2 = \frac{\left(2\cdot k+3\right)\left(2k^2+3k+1+6k+9\right)}{3} =$$

$$= \frac{\left(2\cdot k+3\right)\left(2k^2+9k+10\right)}{3} = \frac{\left(2\cdot k+3\right)\left(2\cdot k+5\right)\left(k+2\right)}{3} \Rightarrow True.$$

So, we proved that this formula is true for every nonnegative integer n.

<u>Answer:</u> Proved.