Question #15309 Find radius of curvature at point (s, ψ) of the curve $s = a \log \tan(\pi/4 + \psi/4)$. .

Solution. It is impossible to calculate the radius of curvature at any point (s, ψ), since tan could be negative. The formula to calculate radius of curvature(in polar coordinates)

$$R = \frac{(r^2 + r_\psi^2)^{3/2}}{|r^2 + 2r_\psi^2 - r r_{\psi\psi}|}$$

Here $r_\psi = r'_\psi$. $r_\psi = a \frac{1}{\tan(\pi/4 + \psi/4)} \frac{1}{\cos^2(\pi/4 + \psi)}$, $r'' = a/16(sec^2(\pi/4 + \psi/4) - csc^2(\pi/4 + \psi/4))$. If you want to evaluate at some point. First evaluate $r, r_\psi, r_{\psi,\psi}$ and put it to the original equation. We took, for instance, $\psi = \pi/3$ and got $2\sqrt{\frac{a^2+3}{9\left|\frac{a}{(a^2+3)^{3/2}}\right|^2 + 3|\frac{a^2}{(a^2+3)^{3/2}}|^2}}$.