Question #51211 Suppose that a computer is connected to a local network. To send a message the computer is consuming the network for a time frame of a multi seconds exactly. With probability p = 0.8, the transmission is a success but , if the transmission does not work then the computer will make a new attempt to transmit after a random period of time has elapsed, and so on until the transmission has been successfully allocated. 1)What is the expected total time required to transmit a message if the random delay has a hope than 3 milliseconds? 2)What is the standard deviation of the total transmission time if the time between transmissions is an integer of milliseconds randomly selected between 1 and 5 inclusive (equal opportunities?).

Solution. 1) Here we are at the situation, either we a successful transmission immediately (this happens with the probability 0.8) or wait 3 seconds for the another attempt and then have a successful attempt (this happens with the probability $0.2 \cdot 0.8$) ans so on. So, in fact, we have that the waiting time has the distribution of 3ξ, where $\xi \sim Geom(0.2)$, that is $\mathsf{P}(\xi = k) = 0.2^k \cdot 0.8$, $k \geq 0$.Hence $E3\xi = 3 \cdot \dfrac{0.2}{0.8} = 3/4.2)$ Here, we have more complicated situation, if the attempt is not successful, then we wait random time ξ, that is distributed as the following $\mathsf{P}(\xi = 1) = \ldots = \mathsf{P}(\xi = 5) = 1/5$.Denote by $\{\xi_i\}_{i\geq 1}$ the sequence of iid r.v. that have the same distribution as ξ, let $\eta \sim Geom(0.2)$ be the r.v., that indicates the number of step when the attempt to transmit is successful. Then, $T = \sum\limits_{i=1}^{\eta} \xi_i$. It is well-known that $VarT = (E\xi)^2 Var\eta + E\eta Var\xi$. It can be easily calculated that $E\eta = 1/4, E\eta^2 = 15/4, Var\eta = 59/16$ and $E\xi = 3, E\xi^2 = 2, Var\xi = 2$. So, $VarT = 539/16 \approx 33.6875$.