

Form the differential equation of which the given function is a solution
 $x^2 + y^2 + 2gx + 2fy + c$.

Solution:

$$u(y)dy = v(x)dx$$

$$u(y) = 2y + 2f$$

$$v(x) = -2x - 2g$$

$$(2y + 2f)dy = -(2x + 2g)dx$$

$$\int (2y + 2f)dy = - \int (2x + 2g)dx$$

$$y^2 + 2fy = -x^2 - 2gx - c,$$

where $c = \text{const.}$

So, the differential equation is:

$$(2y + 2f)dy = -(2x + 2g)dx$$

$$y' = -\frac{x + g}{y + f}$$

Answer: $y' = -\frac{x+g}{y+f}$.