Question 1. *Find* 2 *functions* f *and* g *such that the limit as* x *approaches* 0 *of* $f(x)$ *and the limit as* x *approaches* 0 *of* $g(x)$ *do not exist, but the limit as* x *approaches* 0 *of* $f(x)+g(x)$ *does exist.*

Solution. Simply take $f(x)$, such that $\lim\limits_{x\to 0} f(x)$ does not exist, and set $g(x) = -f(x)$. Then $\lim\limits_{x\to 0} g(x) = \lim\limits_{x\to 0}(-f(x))$ also does not exist, since otherwise it would imply that there is $\lim\limits_{x\to 0} f(x) = -\lim\limits_{x\to 0} g(x)$. Furthermore, $f(x)+g(x) = 0$, so $\lim\limits_{x\to 0}(f(x)+g(x)) = 0$. For example, one can consider $f(x) = \sin(1/x)$. If $x = \frac{1}{\pi n}$, $n \in \mathbb{Z}$, then $f(x) = 0$, but if $x = \frac{1}{\pi/2+2\pi n}$, $n \in \mathbb{Z}$, then $f(x) = 1$, so $\lim\limits_{x\to 0} f(x)$ is not defined. $\square$