Question 1. *Prove that if elements a, b of group G commute, then LCM of their orders is multiple of order of their product.*

Solution. Suppose a and b have finite orders, say m and n, respectively (for the infinite orders this statement does not make sense). Set $k = LCM(m, n)$. Since k is divisible by m and n, we conclude that

$$a^k = (a^m)^{\frac{k}{m}} = e^{\frac{k}{m}} = e, \quad b^k = (b^n)^{\frac{k}{n}} = e^{\frac{k}{n}} = e,$$

where e denotes the identity of G. We are given that a and b commute, therefore

$$(ab)^k = a^k b^k = e \cdot e = e.$$

Thus, the order of ab divides k, i. e. k is a multiple of the order of ab. □